Shift Symmetries, Equivalence of EFTs and Time-Like Extra Dimensions

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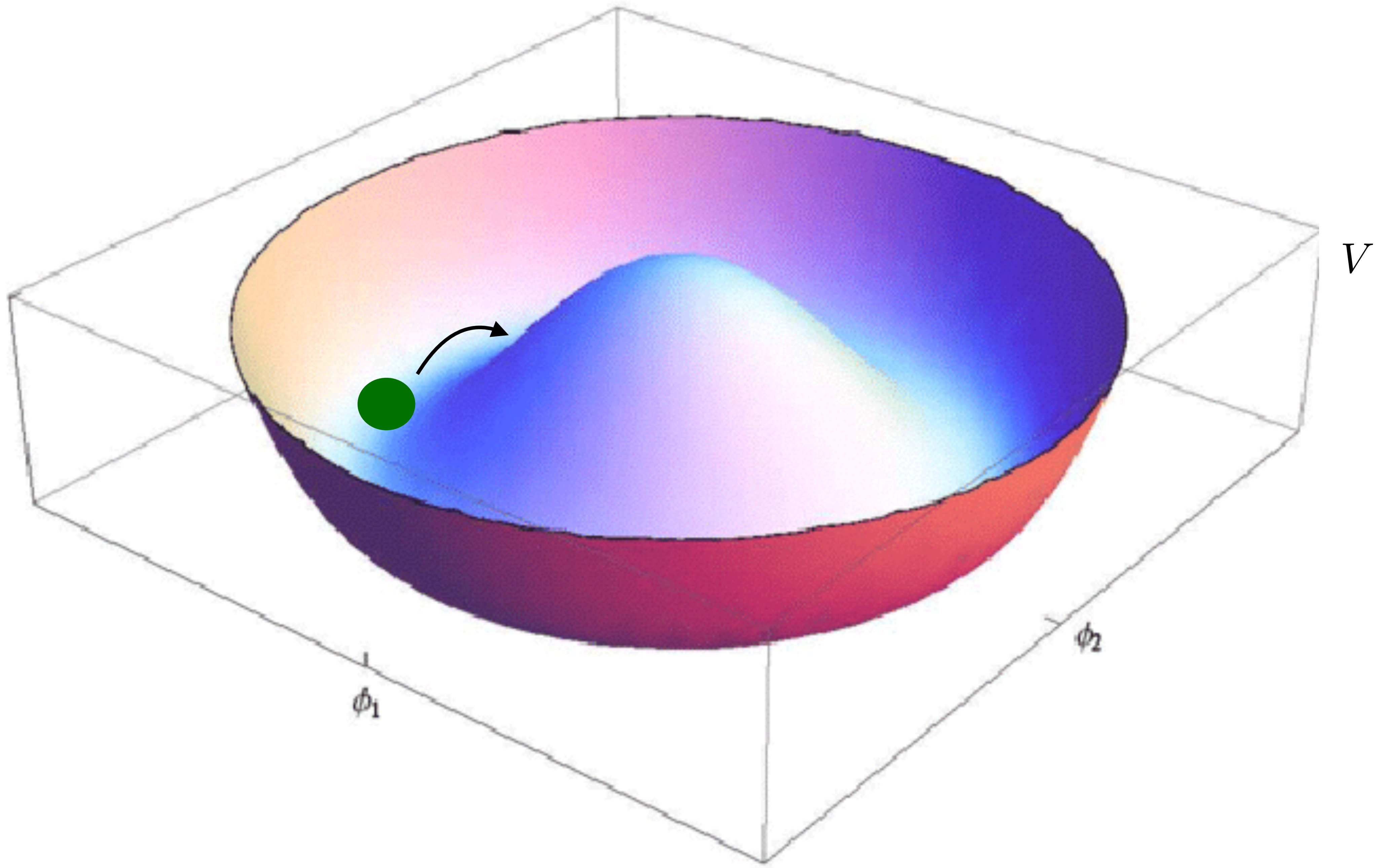
Broken symmetries

Spontaneously broken symmetries → Goldstone Bosons



Goldstone Bosons have shift symmetry

$$\delta\phi = c + \cdots$$

Shift symmetry

A broken symmetry transformation starts with a field-independent term:

$$\delta\phi = c + \mathcal{O}(\phi) + \mathcal{O}(\phi^2) + \cdots$$

Broken symmetry (does not preserve vacuum $\phi = 0$)

$$\delta\phi = \mathcal{O}(\phi) + \mathcal{O}(\phi^2) + \cdots$$

Unbroken symmetry (preserves vacuum $\phi = 0$)

Shift invariant Lagrangian: $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \cdots$

Interactions for an exact shift symmetry: $\delta\phi = c$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + F(\partial\phi, \partial\partial\phi, \cdots) + \lambda\,\phi$$

Function of invariant building block $\partial_\mu\phi$

Wess-Zumino term

Spacetime shift symmetries?

Can shift symmetries/broken symmetries involve spacetime symmetries?

Coleman-Mandula theorem: symmetries of the S-matrix can only be $(\text{Poincare}) \otimes (\text{compact internal})$

Does not forbid *broken* symmetries, which are not ordinary S-matrix symmetries: instead lead to soft-theorems (i.e. Adler zero in pion physics)

Can we classify the possibilities for broken spacetime symmetries?

Galileon symmetry

Scalar kinetic term also has *galileon* symmetry:

spacetime coordinates

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 \quad , \qquad \delta\phi = b_\mu x^\mu$$

constant vector

Boring interactions: $F(\partial\partial\phi, \partial\partial\partial\phi, \cdots)$

Function of invariant
building block $\Pi_{\mu\nu} = \partial_\mu\partial_\nu\phi$

Wess-Zumino terms (*galileons*):

Luty, Porrati, Rattazzi (2003)

Nicolis, Rattazzi, Trincherini (2008)

Garrett Goon, KH, Austin Joyce, Mark Trodden (2012)

$$\mathcal{L}_1 = \phi \ ,$$

$$\mathcal{L}_2 = -\frac{1}{2}(\partial\phi)^2 \ ,$$

$$\mathcal{L}_3 = -\frac{1}{2}(\partial\phi)^2[\Pi] \ ,$$

$$\mathcal{L}_4 = -\frac{1}{2}(\partial\phi)^2\left([\Pi]^2 - [\Pi^2]\right) \ ,$$

$$\mathcal{L}_5 = -\frac{1}{2}(\partial\phi)^2\left([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]\right)$$

DBI symmetry

Deformations of galileon symmetry:

$$\left\{\begin{array}{l} \delta\phi = b_\mu x^\mu + \frac{1}{\Lambda^4} b^\mu \phi\, \partial_\mu \phi \quad \Rightarrow \quad \text{DBI theory} \quad \mathcal{L} = -\Lambda^4 \sqrt{1 + \frac{1}{\Lambda^4}(\partial\phi)^2} \\[2em] \text{Algebra of symmetries:} \quad \mathfrak{iso}(1,4) \ \text{ or } \ \mathfrak{iso}(2,3) \end{array}\right.$$

$$\left\{\begin{array}{l} \delta\phi = b_\mu x^\mu + \frac{1}{\Lambda}\left(b_\nu x^\nu x^\mu \partial_\mu - \frac{1}{2} x^2 b^\mu \partial_\mu\right)\phi \quad \Rightarrow \quad \text{AdS DBI theory} \quad \mathcal{L} = -\Lambda^4 e^{4\phi/\Lambda} \sqrt{1 + \frac{1}{\Lambda^4} e^{-2\phi/\Lambda}(\partial\phi)^2} \\[2em] \text{Algebra of symmetries:} \quad \mathfrak{so}(2,4) \end{array}\right.$$

Extensions of Galileon symmetry

Scalar kinetic term also has *extended galileon* symmetry:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 \quad , \qquad \delta\phi = s_{\mu\nu}x^\mu x^\nu$$

symmetric, traceless constant tensor

special galileon:

Clifford Cheung, Karol Kampf, Jiri Novotny, Jaroslav Trnka (2014)

KH, Austin Joyce (2015)

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{12\Lambda^6}(\partial\phi)^2\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right]$$

$$\delta\phi = s_{\mu\nu}x^\mu x^\nu + \frac{1}{\Lambda^6}s^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

nonlinear deformation of the symmetry

Algebra of symmetries: contraction of $sl(6)$

Extensions of Galileon symmetry

Scalar kinetic term has extended galileon symmetry of all orders:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2$$

$$\delta\phi = c + c_\mu x^\mu + c_{\mu_1\mu_2}x^{\mu_1}x^{\mu_2} + c_{\mu_1\mu_2\mu_3}x^{\mu_1}x^{\mu_2}x^{\mu_3} + \cdots$$

symmetric, traceless constant tensors

There do not seem to be interesting theories at higher orders.

KH, Austin Joyce (2014)

Clifford Cheung, Karol Kampf, Jiri Novotny, Chia-Hsien Shen, Jaroslav Trnka (2016)

Mark Bogers, Tomas Brauner (2018)

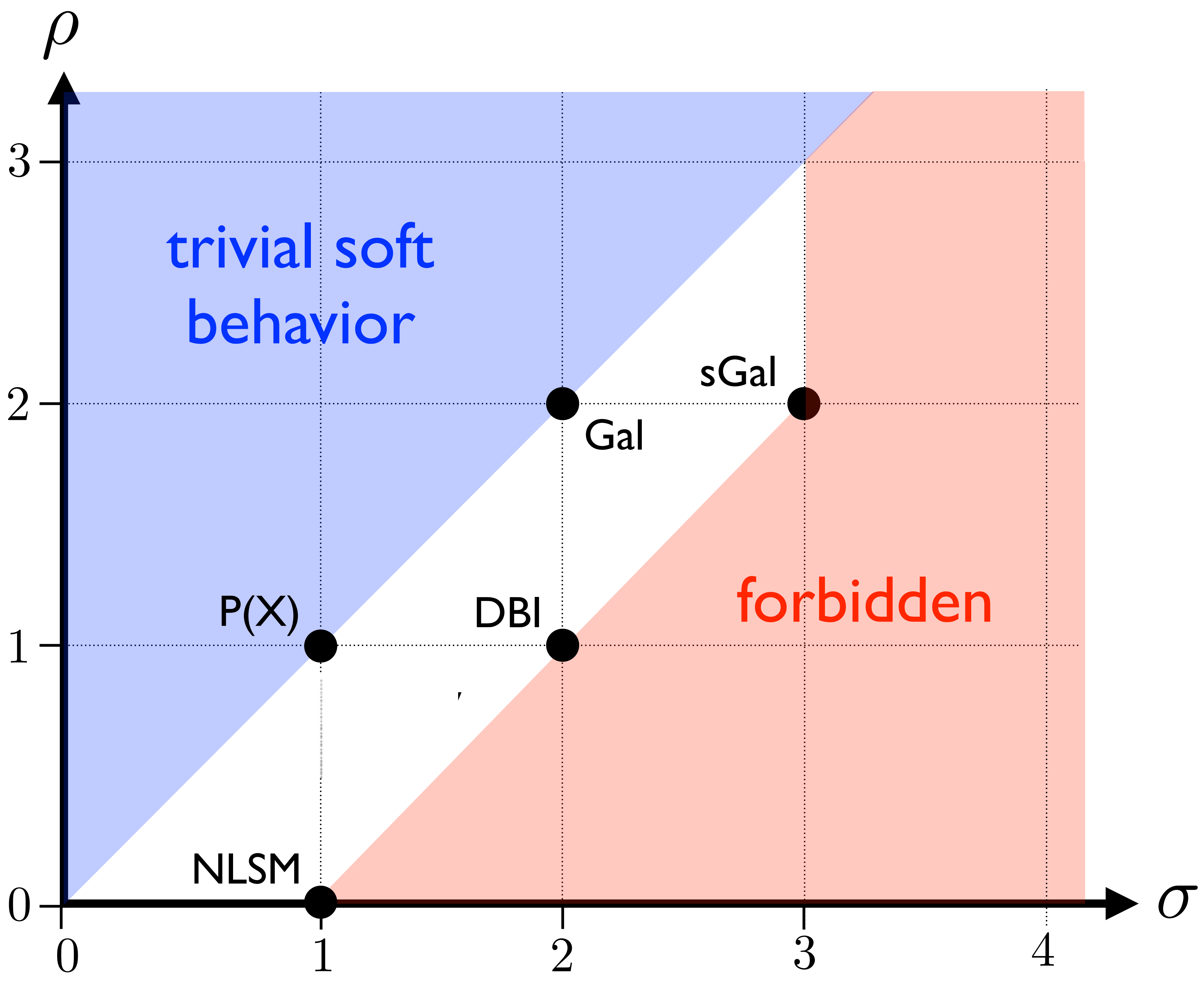
Diederik Roest, David Stefanyszyn, Pelle Werkman (2019)

Classification via S-matrix

Clifford Cheung, Karol Kampf, Jiri Novotny, Chia-Hsien Shen, Jaroslav Trnka (2016)

derivatives per field

$$\mathcal{L} = \partial^2 \phi^2 F(\partial^\rho \phi)$$



Soft limit scaling

$$\lim_{p \to 0} A(p) = \mathcal{O}(p^\sigma)$$

DBI symmetry

Deformations of galileon symmetry:

$\delta\phi = b_\mu x^\mu + \frac{1}{\Lambda^4} b^\mu \phi\, \partial_\mu \phi$ ➡ flat DBI theory $\mathcal{L} = -\Lambda^4 \sqrt{1 + \frac{1}{\Lambda^4} (\partial\phi)^2}$

Algebra of symmetries: $\mathfrak{iso}(1,4)$ or $\mathfrak{iso}(2,3)$

$\delta\phi = b_\mu x^\mu + \frac{1}{\Lambda} \left(b_\nu x^\nu x^\mu \partial_\mu - \frac{1}{2} x^2 b^\mu \partial_\mu \right) \phi$ ➡ AdS DBI theory $\mathcal{L} = -\Lambda^4 e^{4\phi/\Lambda} \sqrt{1 + \frac{1}{\Lambda^4} e^{-2\phi/\Lambda} (\partial\phi)^2}$

Algebra of symmetries: $\mathfrak{so}(2,4)$

First deformation: flat DBI theory

$$\mathcal{L} = -\Lambda^4 \sqrt{1 + \frac{1}{\Lambda^4}(\partial\phi)^2}$$

Symmetries:

unbroken $\left\{ \right.$ D=4 Poincare: $J_{\mu\nu}$, P_μ

broken $\left\{ \right.$ Shift symmetries:

$C \qquad \delta\phi = 1$

$B_\mu \qquad \delta\phi = b_\mu x^\mu + \frac{1}{\Lambda^4} b^\mu \phi \, \partial_\mu \phi$

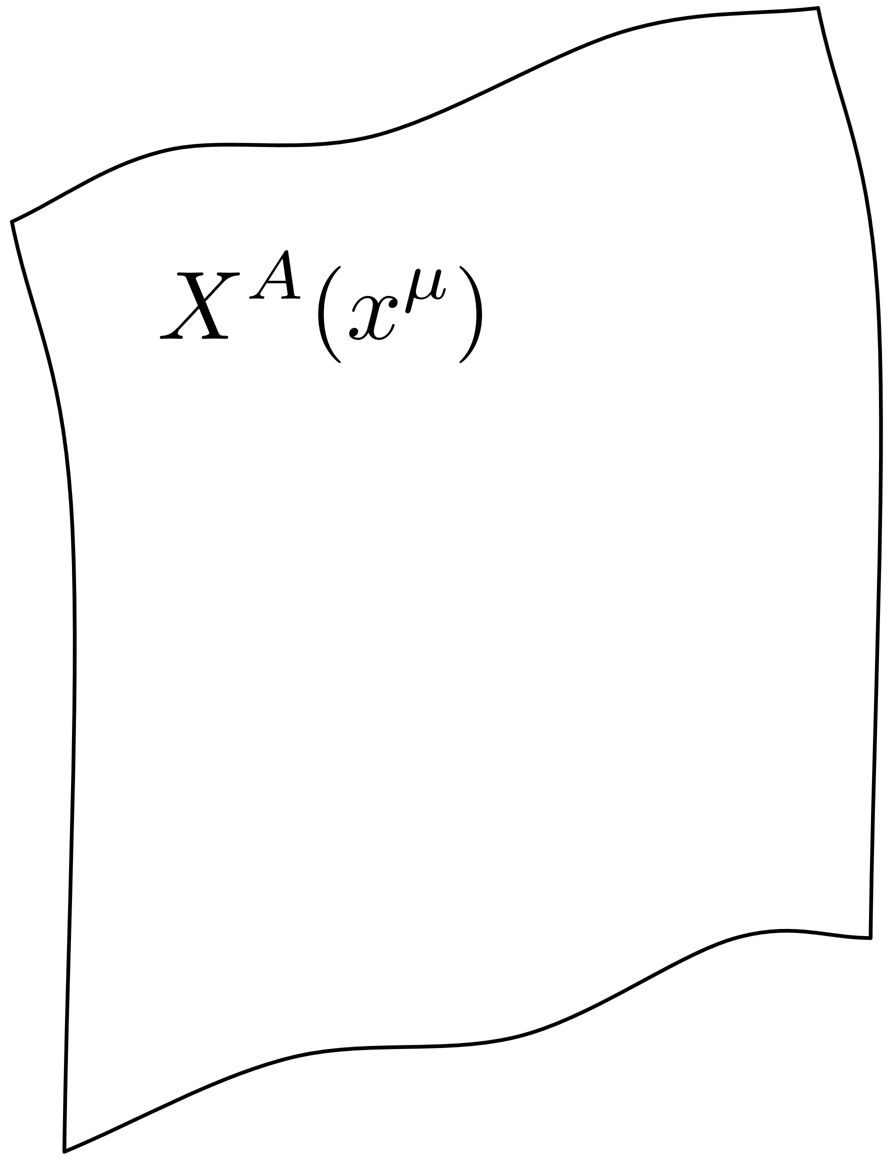
Combines to D=5 poincare:

$$\begin{pmatrix} J_{\mu\nu} & B_\mu \\ -B_\mu & 0 \end{pmatrix} \Rightarrow J_{AB} \qquad\qquad \begin{pmatrix} P_\mu \\ C \end{pmatrix} \Rightarrow P_A$$

Symmetry breaking pattern: $\mathfrak{iso}(1,4) \to \mathfrak{iso}(1,3)$

Probe brane construction

Flat 3-brane embedded in fixed $5D$ Minkowski



$$\eta_{AB} = \begin{pmatrix} -1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & \boxed{1} \end{pmatrix}$$

extra dimension

Action is invariant under bulk Poincare and reparametrizations of the brane worldsheet:

$$\begin{aligned} \delta_P X^A &= \omega^A{}_B X^B + \epsilon^A \\ \delta_g X^A &= \xi^\mu \partial_\mu X^A \end{aligned}$$

Fix isometries with a “unitary” gauge:

$X^\mu(x) = x^\mu, \quad X^5(x) = \phi(x)$

$5D$ poincare

compensating gauge transformation

$$\delta\phi = \delta_P\phi + \delta_g\phi = \underbrace{-\omega^\mu{}_\nu x^\nu \partial_\mu\phi - \epsilon^\mu\partial_\mu\phi}_{\text{4D poincare}} + \underbrace{\omega^5{}_\mu x^\mu - \omega^\mu{}_5 \phi\partial_\mu\phi + \epsilon^5}_{\text{shifts}}$$

Probe brane construction

Actions are constructed from diff invariants of the intrinsic quantities on the brane:

induced metric $g_{\mu\nu} \equiv \frac{\partial X^A}{\partial x^\mu}\frac{\partial X^B}{\partial x^\nu}\eta_{AB}$ $\xrightarrow[\text{gauge } X^\mu(x)=x^\mu]{}$ $g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu\phi\,\partial_\nu\phi$

extrinsic curvature $K_{\mu\nu}$

covariant derivative ∇_μ

intrinsic curvature $R^\rho{}_{\sigma\mu\nu}$

Gauss-Codazzi equations → only need K : $R_{\mu\nu\rho\sigma} = K_{\mu\rho}K_{\nu\sigma} - K_{\mu\sigma}K_{\nu\rho} + \text{bulk curvature}$

Most general invariant Lagrangian: $S = \int d^4x\sqrt{-g}\; F(g_{\mu\nu}, \nabla_\mu, K_{\mu\nu})$

Derivative expansion: $F \sim 1 + K + K^2 + K^3 + \nabla K^2 + \cdots$

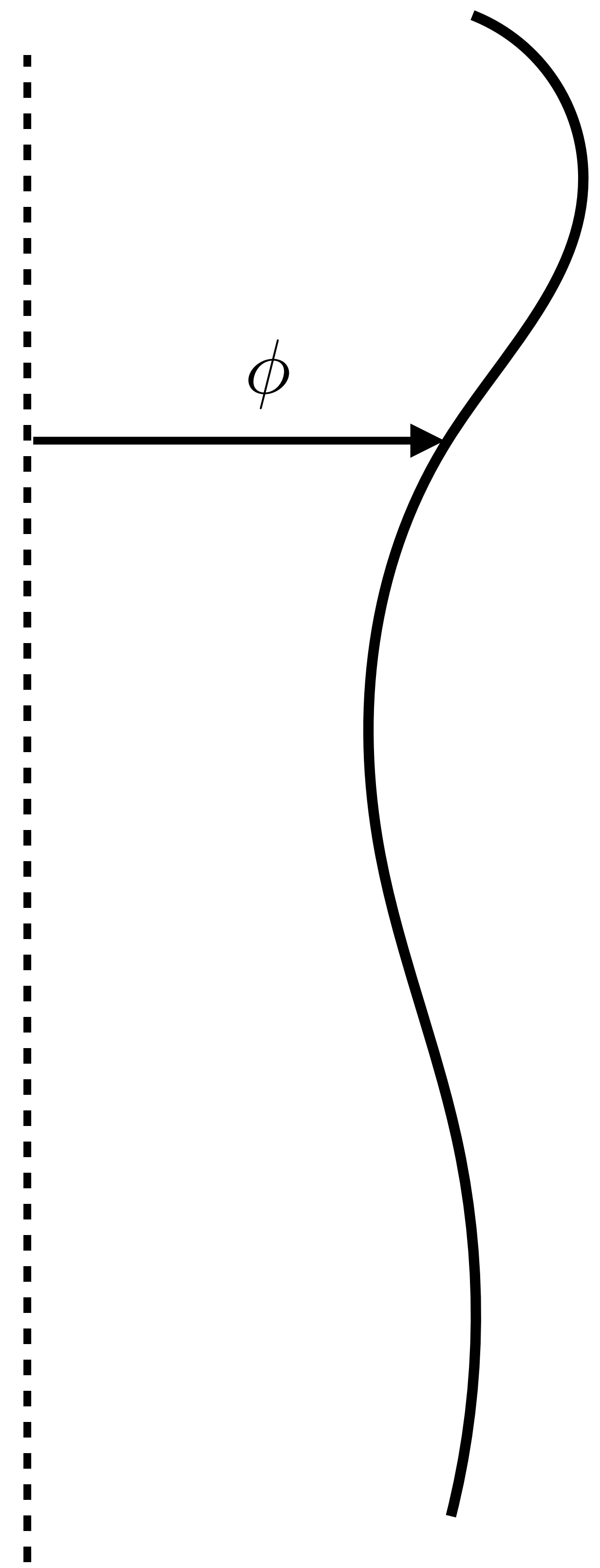
Lowest order: DBI term $\int d^4x\,\sqrt{-g} \to \int d^4x\;\sqrt{1+(\partial\phi)^2}$

Brane construction: tadpole term

One term which can't be written this way (Wess-Zumino term):

5-volume bounded by brane:

$$\mathcal{L}_1 \sim \int^{\phi} d\phi' \sqrt{-G} \sim \phi$$



0 derivative term (tadpole):

must set to zero to have $\phi = 0$ solution

Probe brane construction

At each order: a unique term that gives 2nd order equations of motion: de Rham, Tolley (2010)

Lovelock terms:

$$\mathcal{L}_m = \frac{1}{2^m}\delta^{\alpha_1\beta_1\ldots\alpha_m\beta_m}_{\mu_1\nu_1\ldots\mu_m\nu_m} R_{\alpha_1\beta_1}{}^{\mu_1\nu_1} \ldots . R_{\alpha_m\beta_m}{}^{\mu_m\nu_m}$$

$$\mathcal{L}_0 = 1$$
$$\mathcal{L}_1 = R$$
$$\mathcal{L}_2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$$
$$\begin{aligned}\mathcal{L}_3 \;=\; & 2R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\rho\tau}R^{\rho\tau}{}_{\mu\nu} + 8R^{\mu\nu}{}_{\sigma\rho}R^{\sigma\kappa}{}_{\nu\tau}R^{\rho\tau}{}_{\mu\kappa} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\nu\rho}R^{\rho}{}_{\mu} \\ & +3RR^{\mu\nu\sigma\kappa}R_{\sigma\kappa\mu\nu} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\mu}R_{\kappa\nu} + 16R^{\mu\nu}R_{\nu\sigma}R^{\sigma}{}_{\mu} - 12RR^{\mu\nu}R_{\mu\nu} + R^3\end{aligned}$$

They have Gibbons-Hawking type boundary terms:

$$\begin{aligned}\mathcal{B}_0 &= 0 \\ \mathcal{B}_1 &= K \\ \mathcal{B}_2 &= -\frac{2}{3}K^3_{\mu\nu} + KK^2_{\mu\nu} - \frac{1}{3}K^3 - 2(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})K^{\mu\nu} \\ &\vdots\end{aligned}$$

- In dimension $2m$, integral is a topological invariant (gives the Euler number of the manifold) $\Rightarrow$ total derivative in $2m$ dimensions.
- Gives second order equations in higher dimensions, vanishes identically in lower dimensions (does not add new DOF).

DBI galileons

Terms in D=4:

$$\begin{aligned}\mathcal{L}_2 &= \sqrt{-g}\\ \mathcal{L}_3 &= \sqrt{-g}K\\ \mathcal{L}_4 &= \sqrt{-g}\,R\\ \mathcal{L}_5 &= \sqrt{-g}\,\mathcal{K}_{GB}\end{aligned}$$

Bulk Einstein-Hilbert

Bulk Gauss-Bonnet

Symmetric polynomials in terms of K :

$$S_n[M] = M^{[\mu_1}_{\mu_1} \cdots M^{\mu_n]}_{\mu_n}$$

$$\mathcal{L}_2 \sim \sqrt{-g}S_0[K]$$

$$\mathcal{L}_3 \sim \sqrt{-g}S_1[K]$$

$$\mathcal{L}_4 \sim \sqrt{-g}S_2[K]$$

$$\mathcal{L}_3 \sim \sqrt{-g}S_3[K]$$

DBI galileons

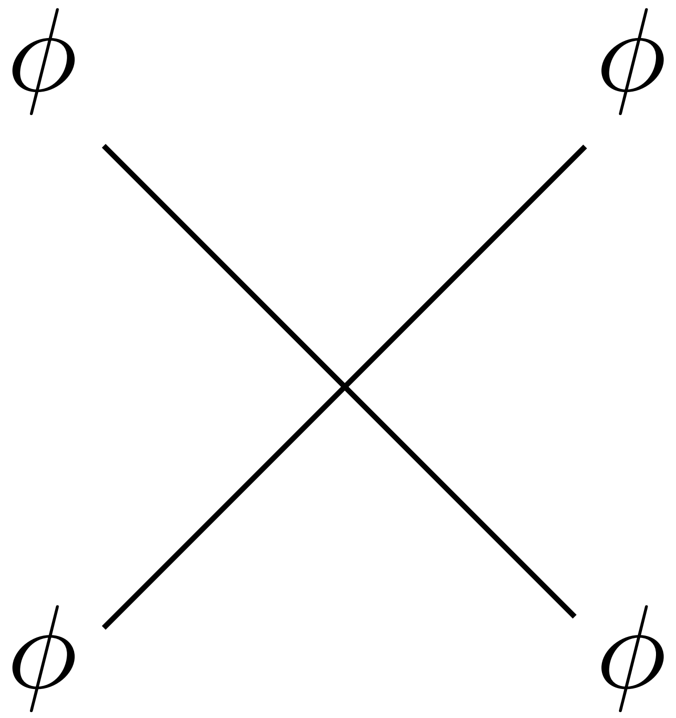
$$\begin{aligned}\mathcal{L}_2 &= -\sqrt{1+(\partial\pi)^2}\\ \mathcal{L}_3 &= -[\Pi]+\gamma^2\left[\pi^3\right]\\ \mathcal{L}_4 &= -\gamma\left([\Pi]^2-\left[\Pi^2\right]\right)-2\gamma^3\left(\left[\pi^4\right]-[\Pi]\left[\pi^3\right]\right)\\ \mathcal{L}_5 &= -\gamma^2\left([\Pi]^3+2\left[\Pi^3\right]-3\,[\Pi]\left[\Pi^2\right]\right)\\ &\quad -\gamma^4\left(6\,[\Pi]\left[\pi^4\right]-6\left[\pi^5\right]-3\left([\Pi]^2-\left[\Pi^2\right]\right)\left[\pi^3\right]\right)\end{aligned}$$

DBI theory: S-matrix

Lowest order DBI term:

$$\mathcal{L} = -\Lambda^4\sqrt{1+\frac{(\partial\phi)^2}{\Lambda^4}} = const. - \frac{1}{2}(\partial\phi)^2 + \frac{1}{8\Lambda^4}(\partial\phi)^4 + \cdots$$

tree level 4-pt. amplitude



$$\mathcal{A}_4 = \frac{1}{4\Lambda^4}\left(s^2+t^2+u^2\right)$$

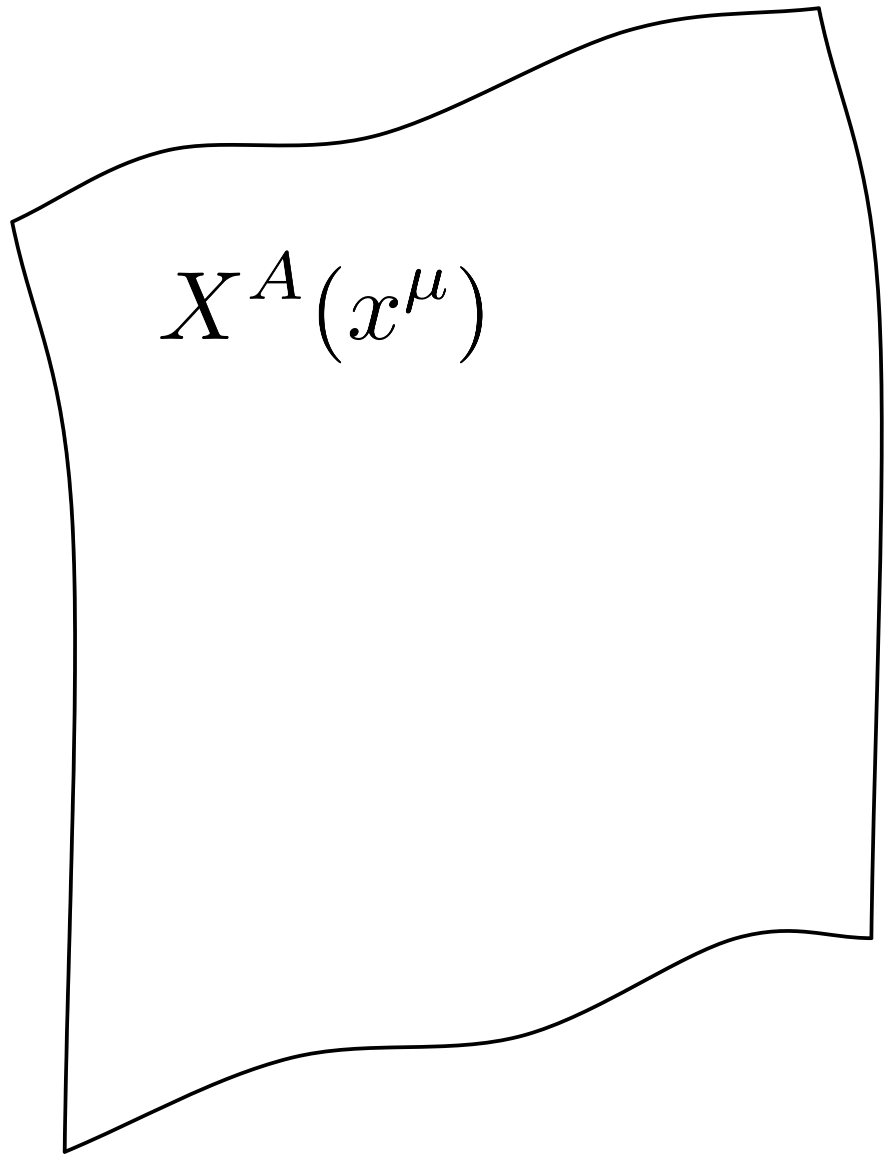
consistent with positivity bounds

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

Wrong sign BI theory

Mukhanov, Vikman (2005)

What if we have a *timelike* extra dimension?



$$\eta_{AB} = \begin{pmatrix} -1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & \boxed{-1} \end{pmatrix}$$

extra dimension

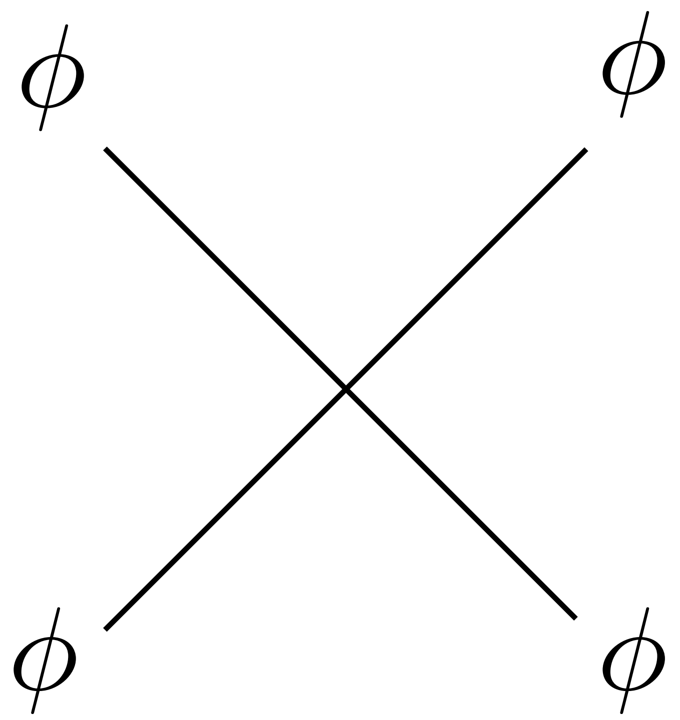
Induced metric: $g_{\mu\nu} = \eta_{\mu\nu} \boxed{-} \partial_\mu \phi \, \partial_\nu \phi$

DBI term: $\int d^4x \, \sqrt{-g} \to \int d^4x \, \sqrt{1 \boxed{-} (\partial\phi)^2}$

Wrong sign DBI theory

$$\mathcal{L} = \Lambda^4 \sqrt{1 - \frac{(\partial\phi)^2}{\Lambda^4}} = const. - \frac{1}{2}(\partial\phi)^2 - \frac{1}{8\Lambda^4}(\partial\phi)^4 + \cdots$$

tree level 4-pt. amplitude



$$\mathcal{A}_4 = \boxed{-}\frac{1}{4\Lambda^4}\left(s^2 + t^2 + u^2\right)$$

violates positivity bounds

Not equivalent to correct-sign DBI theory

Shift symmetries: C $\qquad \delta\phi = 1$

$B_\mu \qquad \delta\phi = b_\mu x^\mu \boxed{-} \frac{1}{\Lambda^4} b^\mu \phi \partial_\mu \phi$

Symmetry breaking pattern:

$$\mathfrak{iso}(2,3) \to \mathfrak{iso}(1,3)$$

Equivalence of EFTs

Landau/Wilson paradigm:

Symmetry breaking pattern + light degrees of freedom → Low energy dynamics

Low energy physics = EFT → same should be true of EFTs

Is it true when space-time symmetries are broken?

DBI symmetry

Deformations of galileon symmetry:

$\delta\phi = b_\mu x^\mu + \frac{1}{\Lambda^4} b^\mu \phi\, \partial_\mu \phi$ → flat DBI theory $\mathcal{L} = -\Lambda^4 \sqrt{1 + \frac{1}{\Lambda^4} (\partial\phi)^2}$

Algebra of symmetries: $\mathfrak{iso}(1,4)$ or $\mathfrak{iso}(2,3)$

$\delta\phi = b_\mu x^\mu + \frac{1}{\Lambda} \left(b_\nu x^\nu x^\mu \partial_\mu - \frac{1}{2} x^2 b^\mu \partial_\mu \right) \phi$ → AdS DBI theory $\mathcal{L} = -\Lambda^4 e^{4\phi/\Lambda} \sqrt{1 + \frac{1}{\Lambda^4} e^{-2\phi/\Lambda} (\partial\phi)^2}$

Algebra of symmetries: $\mathfrak{so}(2,4)$

Other deformation: AdS DBI theory

unbroken
$$\begin{cases} P_\mu \phi = -\partial_\mu \phi \,, \\ J_{\mu\nu}\, \phi = (x_\mu \partial_\nu - x_\nu \partial_\mu)\phi \,, \end{cases}$$

broken
$$\begin{cases} D\phi = -1 - x^\mu \partial_\mu \phi \,, \\ K_\mu \phi = -2x_\mu + \left[-2x_\mu x^\nu \partial_\nu + \left(x^2 + L^2 e^{-2\phi}\right)\partial_\mu\right]\phi \,. \end{cases}$$

$$[D, P_\mu] = -P_\mu, \quad [D, K_\mu] = K_\mu, \quad [K_\mu, P_\nu] = 2J_{\mu\nu} - 2\eta_{\mu\nu} D \,,$$
$$[J_{\mu\nu}, K_\sigma] = \eta_{\mu\sigma} K_\nu - \eta_{\nu\sigma} K_\mu, \quad [J_{\mu\nu}, P_\sigma] = \eta_{\mu\sigma} P_\nu - \eta_{\nu\sigma} P_\mu \,,$$
$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho} - \eta_{\mu\sigma} J_{\nu\rho} \,,$$

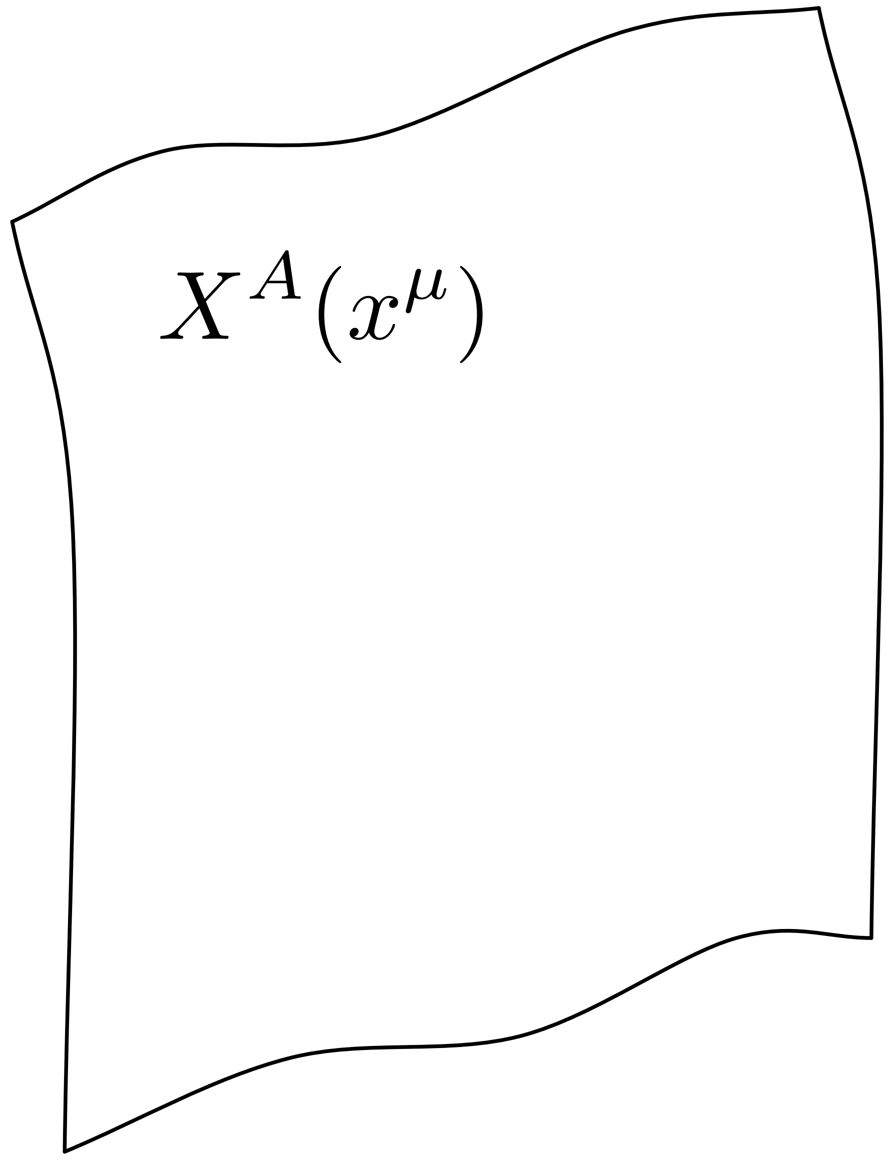
$$\mathbb{J}^{AB} = \left(\begin{array}{c|c|c} 0 & D & \frac{1}{2}\left(P^\nu - K^\nu\right) \\ \hline -D & 0 & \frac{1}{2}\left(P^\nu + K^\nu\right) \\ \hline -\frac{1}{2}\left(P^\mu - K^\mu\right) & -\frac{1}{2}\left(P^\mu + K^\mu\right) & J^{\mu\nu} \end{array}\right)$$

$$[\mathbb{J}^{AB}, \mathbb{J}^{CD}] = \mathbb{G}^{AC}\mathbb{J}^{BD} - \mathbb{G}^{BC}\mathbb{J}^{AD} + \mathbb{G}^{BD}\mathbb{J}^{AC} - \mathbb{G}^{AD}\mathbb{J}^{BC} \,, \qquad \mathbb{G}^{AB} = \left(\begin{array}{c|c|c} -1 & & \\ \hline & 1 & \\ \hline & & \eta^{\mu\nu} \end{array}\right)$$

Symmetry breaking pattern: $\mathfrak{so}(2,4) \rightarrow \mathfrak{iso}(1,3)$

AdS DBI theory

Flat brane in bulk AdS:



$$ds^2 = d\rho^2 + e^{2\rho/L}\eta_{\mu\nu}dx^\mu dx^\nu$$

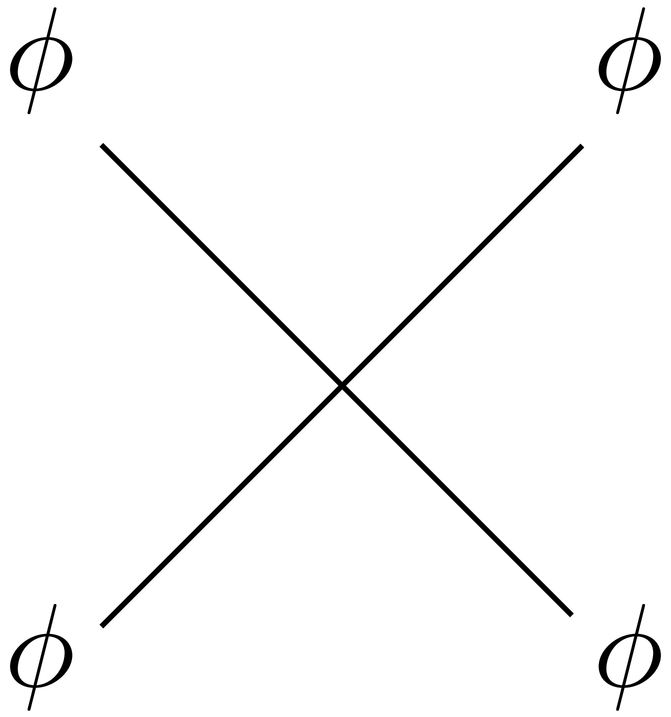
Induced metric: $g_{\mu\nu} = e^{2\phi}\eta_{\mu\nu} + L^2\partial_\mu\phi\partial_\nu\phi$

DBI term: $\int d^4x\,\sqrt{-g} \rightarrow \int d^4x\;\; e^{4\phi}\sqrt{1+e^{-2\phi}L^2(\partial\phi)^2}$

AdS theory: amplitudes

$$\mathcal{L} = \frac{1}{L^4} e^{4L\phi} \left(1 - \sqrt{1 + e^{-2L\phi} L^4 (\partial\phi)^2}\right) = -\frac{1}{2}(\partial\phi)^2 + \frac{L^4}{8}(\partial\phi)^4 + \text{on shell trivial} + \cdots$$

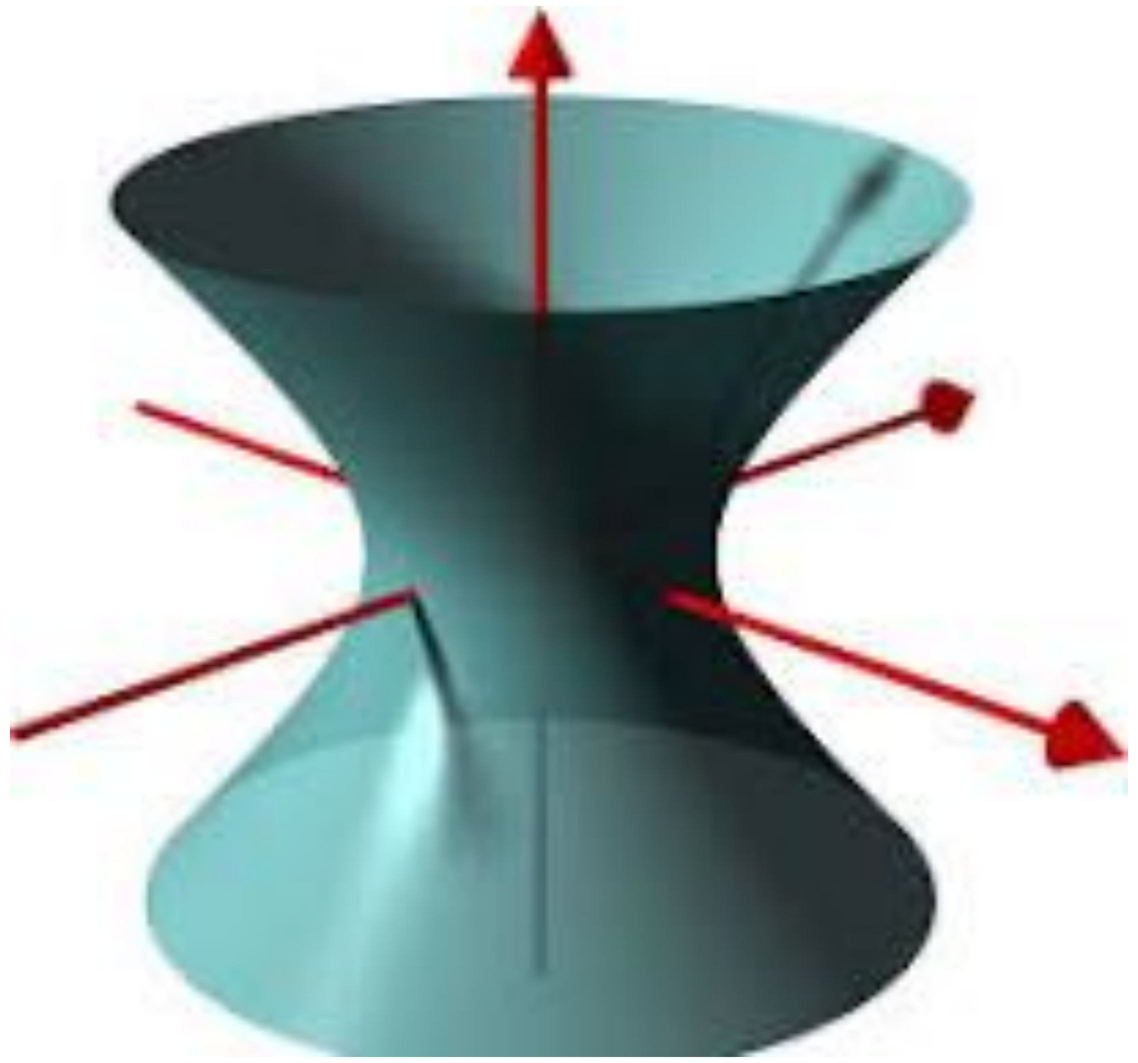
tree level 4-pt. amplitude



$$\mathcal{A}_4 = \frac{L^4}{4}\left(s^2 + t^2 + u^2\right)$$

consistent with positivity bounds

AdS space



$\mathrm{AdS}_{1,4}$ is a hyperbola in $\mathrm{M}_{2,4}$ embedding space:

$$\eta_{AB}Y^AY^B = -L^2 \quad , \qquad \eta_{AB} = \mathrm{diag}(-1,-1,1,1,1,1)$$

Maximally symmetric space with (-,+,+,+,+) signature and $R<0$

Manifests the isometries: $\mathfrak{so}(2,4)$

Poincare coordinates:

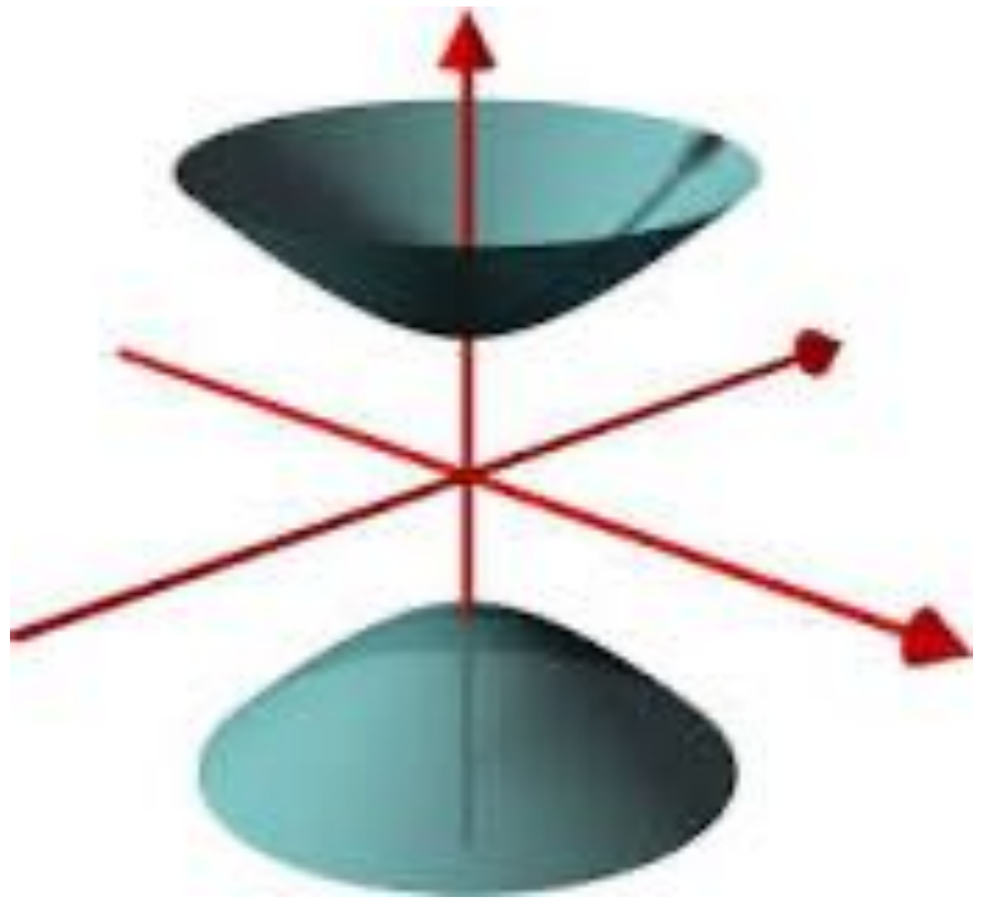
$$Y^0 = L\cosh(\rho/L) + \frac{1}{2L}e^{\rho/L}x^2 \ ,$$
$$Y^1 = e^{\rho/L}x^0 \ ,$$
$$Y^2 = L\sinh(\rho/L) - \frac{1}{2L}e^{\rho/L}x^2 \ ,$$
$$Y^{i+2} = e^{\rho/L}x^i, \qquad i = 1,2,3 \quad .$$

Induced metric: $$ds^2 = d\rho^2 + e^{2\rho/L}\eta_{\mu\nu}dx^\mu dx^\nu$$

Manifests unbroken subgroup: $\mathfrak{iso}(1,3)$

2-time dS space

dS$_{2,3}$ is the other hyperbola in M$_{2,4}$ embedding space:



$$\eta_{AB} Y^A Y^B = +L^2 \quad , \qquad \eta_{AB} = \mathrm{diag}(-1,-1,1,1,1,1)$$

Maximally symmetric space with (-,-,+,+,+) signature and R>0

Same isometries: $\mathfrak{so}(2,4)$

Poincare coordinates:

$$\begin{aligned} Y^0 &= L\sinh(\rho/L) + \frac{1}{2L} e^{\rho/L} x^2 \ , \\ Y^1 &= e^{\rho/L} x^0 \ , \\ Y^2 &= L\cosh(\rho/L) - \frac{1}{2L} e^{\rho/L} x^2 \ , \\ Y^{i+2} &= e^{\rho/L} x^i \ , \qquad i = 1,2,3 \qquad , \end{aligned}$$

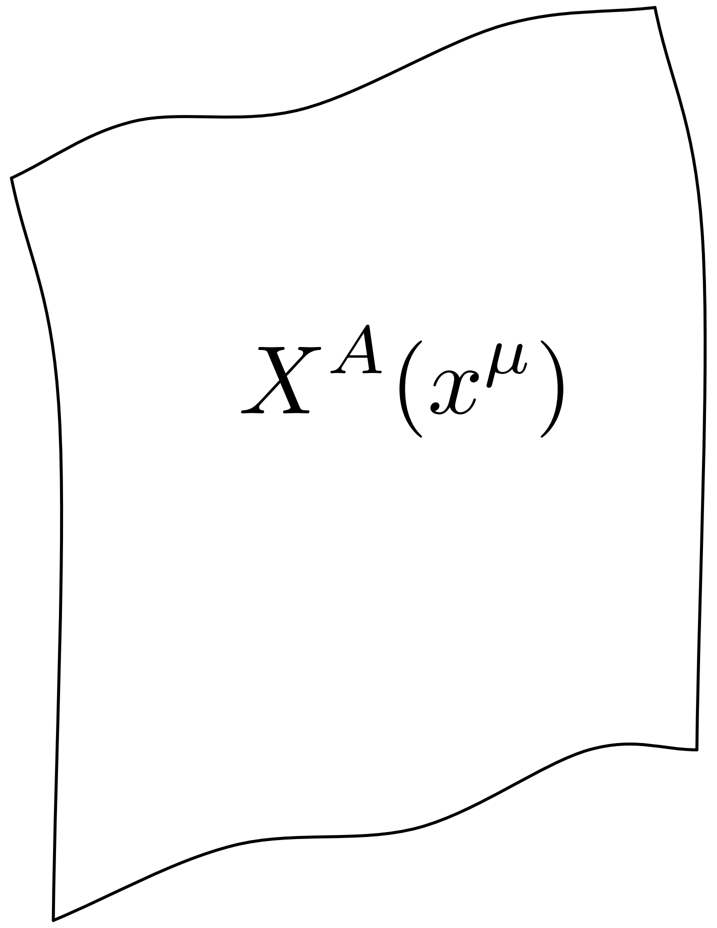
Induced metric:

$$ds^2 = \boxed{-} d\rho^2 + e^{2\rho/L} \eta_{\mu\nu} dx^\mu dx^\nu$$

Manifests unbroken subgroup: $\mathfrak{iso}(1,3)$

2-time dS theory

Flat brane in bulk $\mathrm{dS}_{2,3}$:



$$ds^2 = -d\rho^2 + e^{2\rho/L}\eta_{\mu\nu}dx^\mu dx^\nu$$

Induced metric: $g_{\mu\nu} = e^{2\phi}\eta_{\mu\nu} - L^2\partial_\mu\phi\partial_\nu\phi$

Different symmetry transformations:

$$\text{unbroken}\begin{cases} P_\mu\,\phi = -\partial_\mu\phi, \\ J_{\mu\nu}\,\phi = (x_\mu\partial_\nu - x_\nu\partial_\mu)\phi\,, \end{cases}$$

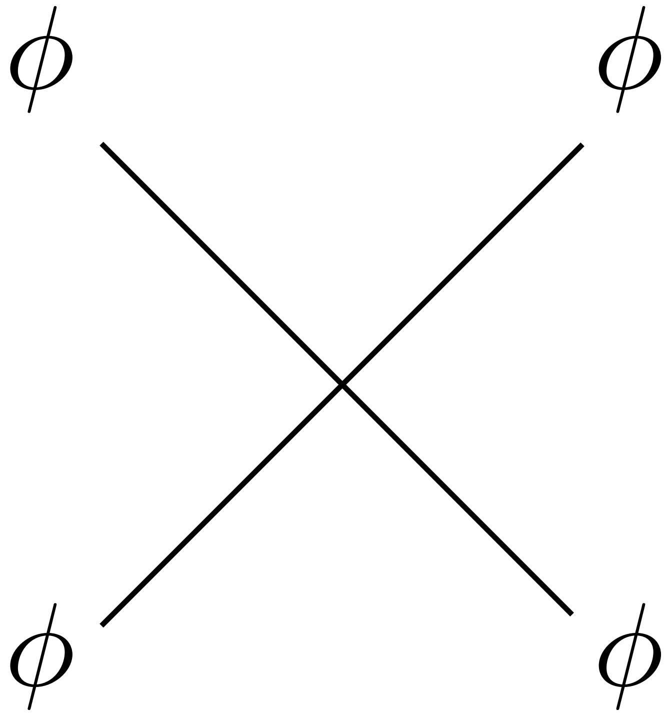
$$\text{broken}\begin{cases} D\phi = -1 - x^\mu\partial_\mu\phi, \\ K_\mu\,\phi = -2x_\mu + \left(-2x_\mu x^\nu\partial_\nu + \left(x^2 \boxed{-} L^2 e^{-2\phi}\right)\partial_\mu\right)\phi \end{cases}$$

Commutators unchanged → *Same* symmetry breaking pattern: $\mathfrak{so}(2,4) \to \mathfrak{iso}(1,3)$

2-time dS theory

DBI term: $\mathcal{L} = -\frac{1}{L^4}e^{4L\phi}\left(1-\sqrt{1\boxed{-}e^{-2L\phi}L^4(\partial\phi)^2}\right) = -\frac{1}{2}(\partial\phi)^2\boxed{-}\frac{L^4}{8}(\partial\phi)^4 + \text{on shell trivial} + \cdots$

tree level 4-pt. amplitude:



$$\mathcal{A}_4 = \boxed{-}\frac{L^4}{4}\left(s^2+t^2+u^2\right)$$

violates positivity bounds

We seem to have 2 EFTs with same symmetry breaking pattern and seemingly different amplitudes

Is this a counterexample to (symmetries)+(degrees of freedom) → (theory) ?

Coset construction

Callan, Coleman, Wess, Zumino, (1969)

Symmetry breaking pattern + degrees of freedom → Lagrangian

Full symmetry group: G

unbroken subgroup: H

Goldstone bosons: G/H

Basis of G: $\{V_I, Z_a\}$

unbroken

broken

Coset element: $$V(x) = e^{\xi(x)\cdot Z}$$

Maurer-Cartan form: $$V^{-1}\mathrm{d}V = \omega_V^I V_I + \omega_Z^a Z_a$$

invariant connection

invariant building block

Invariant Lagrangian: $$\mathcal{L}(\omega_Z, \omega_V)$$ + Wess-Zumino terms

Invariant up to total derivative, classified by Lie algebra cohomology

Coset construction for spacetime symmetries

Volkov, Ogievetsky (1973)

Full symmetry group: G
unbroken subgroup: H
Goldstone bosons: G/H

broken

Basis of G: $\{V_I, Z_a\}$

unbroken (includes translations P, rotations J)

spacetime coordinates

Coset element: $$V = e^{x\cdot P} e^{\xi(x)\cdot Z}$$

Maurer-Cartan form: $$V^{-1}\mathrm{d}V = \omega_P^\alpha P_\alpha + \omega_Z^a Z_a + \omega_V^I V_I + \frac{1}{2}\omega_J^{\alpha\beta} J_{\alpha\beta}$$

invariant vielbein

invariant building block

invariant connection

More broken generators than Goldstones → Inverse Higgs constraints:

$[P, Z_1] \sim Z_2 + \cdots$ ➡ can eliminate Z_1 component in MC form

Weyl theory

$SO(2,4)$ conformal algebra: $$\{\underbrace{K_\mu, D}_{\text{broken}}, \underbrace{P_\mu, J_{\mu\nu}}_{\text{unbroken}}\}$$

$\mathfrak{so}(2,4) \to \mathfrak{iso}(1,3)$

$$[D, P_\mu] = -P_\mu, \quad [D, K_\mu] = K_\mu, \quad [K_\mu, P_\nu] = 2J_{\mu\nu} - 2\eta_{\mu\nu}D\,,$$
$$[J_{\mu\nu}, K_\sigma] = \eta_{\mu\sigma}K_\nu - \eta_{\nu\sigma}K_\mu, \quad [J_{\mu\nu}, P_\sigma] = \eta_{\mu\sigma}P_\nu - \eta_{\nu\sigma}P_\mu\,,$$
$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho} - \eta_{\mu\sigma}J_{\nu\rho}\,,$$

Maurer-Cartan form: $$V = e^{y\cdot P}e^{\pi D}e^{\xi\cdot K} \quad , \qquad V^{-1}dV = \omega_P^\alpha P_\alpha + \omega_D D + \omega_K^\alpha K_\alpha + \frac{1}{2}\omega_J^{\alpha\beta}J_{\alpha\beta}$$

$$\omega_P^\alpha = e^\pi \mathrm{d}y^\alpha\,,$$
$$\omega_D = \mathrm{d}\pi + 2e^\pi\xi_\mu \mathrm{d}y^\mu\,,$$
$$\omega_K^\alpha = \mathrm{d}\xi^\alpha + \xi^\alpha\mathrm{d}\pi + e^\pi\left(2\xi^\alpha\xi_\nu\mathrm{d}y^\nu - \xi^2\mathrm{d}y^\alpha\right)\,,$$
$$\omega_J^{\alpha\beta} = -4e^\pi\xi^{[\alpha}\mathrm{d}y^{\beta]}\,,$$

Symmetry transformations:

$$P_\mu\,\pi = -\partial_\mu\pi,$$
$$J_{\mu\nu}\,\pi = (x_\mu\partial_\nu - x_\nu\partial_\mu)\pi\,,$$
$$D\pi = -1 - x^\mu\partial_\mu\pi,$$
$$K_\mu\,\pi = -2x_\mu + \left(-2x_\mu x^\nu\partial_\nu + x^2\partial_\mu\right)\pi$$

Weyl theory

Inverse Higgs constraint:

$$\begin{aligned}
\omega_P^\alpha &= e^\pi \mathrm{d}y^\alpha \,, \\
&\boxed{\omega_D = \mathrm{d}\pi + 2e^\pi \xi_\mu \mathrm{d}y^\mu \,,} \\
\omega_K^\alpha &= \mathrm{d}\xi^\alpha + \xi^\alpha \mathrm{d}\pi + e^\pi \left(2\xi^\alpha \xi_\nu \mathrm{d}y^\nu - \xi^2 \mathrm{d}y^\alpha\right) \,, \\
\omega_J^{\alpha\beta} &= -4e^\pi \xi^{[\alpha} \mathrm{d}y^{\beta]} \,,
\end{aligned}$$

$$\xrightarrow{\xi_\mu = -\frac{1}{2}e^{-\pi}\partial_\mu \pi}$$

$$\begin{aligned}
\omega_P^\alpha &= e^\pi \mathrm{d}y^\alpha \,, \\
\omega_K^\alpha &= \tfrac{1}{2}e^{-\pi} \left(\partial_\nu \pi \partial^\alpha \pi - \partial_\nu \partial^\alpha \pi - \tfrac{1}{2}(\partial\pi)^2 \delta_\nu^\alpha\right) \mathrm{d}y^\nu \\
\omega_J^{\alpha\beta} &= 2\partial^{[\alpha}\pi \mathrm{d}y^{\beta]} \,.
\end{aligned}$$

invariant metric: $g_{\mu\nu} = e^{2\pi}\eta_{\mu\nu}$

→ invariant building block: $\mathcal{D}_\mu \xi_\nu = \frac{1}{2}\partial_\mu \pi \partial_\nu \pi - \frac{1}{2}\partial_\mu \partial_\nu \pi - \frac{1}{4}(\partial\pi)^2 \eta_{\mu\nu}$

$$R_{\mu\nu}(g) = 2\partial_\mu \pi \partial_\nu \pi - 2\partial_\mu \partial_\nu \pi - \Box\pi \eta_{\mu\nu} - 2(\partial\pi)^2 \eta_{\mu\nu} = 4\mathcal{D}_\mu \xi_\nu + 2\mathcal{D}_\rho \xi^\rho g_{\mu\nu}$$

Lagrangian and derivative expansion: $\mathcal{L}(g_{\mu\nu}, R_{\mu\nu}) \sim \sqrt{-g}\left(1 + R + R^2 + \cdots\right)$

Weyl theory

Ghost-free Lagrangians (conformal galileons): Nicolis, Rattazzi, Trincherini (2009)

$\mathcal{L}_1^{(\mathrm{Weyl})}$	$-\frac{1}{4L^4}\sqrt{-g}$	$-\frac{1}{4L^4}\sqrt{-g}$	$-\frac{1}{4L^4}e^{4\pi}$
$\mathcal{L}_2^{(\mathrm{Weyl})}$	$-\frac{1}{12L^2}\sqrt{-g}R$	$-\frac{1}{L^2}\sqrt{-g}S_1[\mathcal{D}\xi]$	$-\frac{1}{2L^2}e^{2\pi}(\partial\pi)^2$
$\mathcal{L}_3^{(\mathrm{Weyl})}$	(Wess-Zumino term)		$\frac{1}{2}(\partial\pi)^2\Box\pi+\frac{1}{4}(\partial\pi)^4$
$\mathcal{L}_4^{(\mathrm{Weyl})}$	$L^2\sqrt{-g}\,\frac{1}{8}\left(\frac{7}{36}[R]^3-[R][R^2]+[R^3]\right)$	$L^2\sqrt{-g}\,24\,S_3[\mathcal{D}\xi]$	$\frac{L^2}{2}e^{-2\pi}(\partial\pi)^2\left[-2S_2[\partial\partial\pi]+\frac{1}{2}(\partial\pi)^2\Box\pi-\frac{1}{2}(\partial\pi)^4\right]$
$\mathcal{L}_5^{(\mathrm{Weyl})}$	$L^4\sqrt{-g}\,\frac{1}{192}\left(\frac{31}{18}[R]^4-13[R^2][R]^2+9[R^2]^2+20[R][R^3]-18[R^4]\right)$	$L^4\sqrt{-g}\,96\,S_4[\mathcal{D}\xi]$	$\frac{L^4}{2}e^{-4\pi}(\partial\pi)^2\left[6S_3[\partial\partial\pi]-6(\partial\pi)^2S_2[\partial\partial\pi]+5(\partial\pi)^4\Box\pi-\frac{14}{4}(\partial\pi)^6\right]$

Homogeneous in derivatives: n-th Lagrangian contains only terms with $2n-2$ derivatives.

AdS theory

Different parametrization of the coset:

$$\hat{K}^{\mu} = K^{\mu} + L^2 P^{\mu} \qquad , \qquad \{\underbrace{\hat{K}_{\mu}, D,}_{\text{unbroken}} \underbrace{P_{\mu}, J_{\mu\nu}}_{\text{broken}}\}$$

$$[D, P_\mu] = -P_\mu, \quad [D, \hat{K}_\mu] = \hat{K}_\mu - 2L^2 P_\mu, \quad [\hat{K}_\mu, P_\nu] = 2J_{\mu\nu} - 2\eta_{\mu\nu} D\,, \quad [\hat{K}_\mu, \hat{K}_\nu] = 4L^2 J_{\mu\nu}\,,$$

$$[J_{\mu\nu}, \hat{K}_\sigma] = \eta_{\mu\sigma}\hat{K}_\nu - \eta_{\nu\sigma}\hat{K}_\mu, \quad [J_{\mu\nu}, P_\sigma] = \eta_{\mu\sigma}P_\nu - \eta_{\nu\sigma}P_\mu\,,$$

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho} - \eta_{\mu\sigma}J_{\nu\rho}\,,$$

Maurer-Cartan form:

$$V = e^{x\cdot P} e^{\phi D} e^{\Lambda\cdot\hat{K}} \qquad , \qquad \omega = V^{-1}dV = \hat{\omega}_P^{\alpha} P_\alpha + \hat{\omega}_D D + \hat{\omega}_{\hat{K}}^{\alpha}\hat{K}_\alpha + \frac{1}{2}\hat{\omega}_J^{\alpha\beta} J_{\alpha\beta}$$

$$P_\mu\,\phi = -\partial_\mu\phi\,,$$

$$J_{\mu\nu}\,\phi = (x_\mu\partial_\nu - x_\nu\partial_\mu)\phi\,,$$

$$D\phi = -1 - x^\mu\partial_\mu\phi\,,$$

$$K_\mu\,\phi = -2x_\mu + \left[-2x_\mu x^\nu\partial_\nu + \left(x^2 + L^2 e^{-2\phi}\right)\partial_\mu\right]\phi$$

AdS theory

Inverse Higgs constraint: $\lambda_\mu = -\frac{e^{-\phi}\partial_\mu\phi}{1+\sqrt{1+L^2e^{-2\phi}(\partial\phi)^2}}$

$$\boxed{\hat{\omega}_D = \frac{1-L^2\lambda^2}{1+L^2\lambda^2}\left(d\phi + 2e^\phi\frac{\lambda_\mu}{1-L^2\lambda^2}dx^\mu\right) ,}$$

$$\lambda^\mu \equiv \frac{1}{L}\Lambda^\mu\frac{\tan(L\Lambda)}{\Lambda}, \quad \Lambda = \sqrt{\Lambda_\mu\Lambda^\mu}$$

$$\hat{\omega}_P^\alpha = -\frac{2L^2}{1+L^2\lambda^2}\lambda^\alpha d\phi + e^\phi\left(dx^\alpha - \frac{2L^2}{1+L^2\lambda^2}\lambda^\alpha\lambda_\mu dx^\mu\right) ,$$

$$\hat{\omega}_{\hat{K}}^\alpha = \frac{1}{1+L^2\lambda^2}\left[d\lambda^\alpha + \lambda^\alpha d\phi + e^\phi\left(-\lambda^2dx^\alpha + 2\lambda^\alpha\lambda_\mu dx^\mu\right)\right] ,$$

$$\hat{\omega}_J^{\alpha\beta} = -\frac{4}{1+L^2\lambda^2}\left(L^2\lambda^{[\alpha}d\lambda^{\beta]} + e^\phi\lambda^{[\alpha}dx^{\beta]}\right) ,$$

invariant metric: $g_{\mu\nu} = e^{2\phi}\eta_{\mu\nu} + L^2\partial_\mu\phi\partial_\nu\phi$

➡

invariant building block: $\mathcal{D}_\mu\Lambda_\nu = \frac{1}{2L}\left(K_{\mu\nu} - \frac{1}{L}g_{\mu\nu}\right)$

$$K_{\mu\nu} = \frac{\gamma}{L}\left(e^{2\phi}\eta_{\mu\nu} - L^2\partial_\mu\partial_\nu\phi + 2L^2\partial_\mu\phi\partial_\nu\phi\right)$$

AdS theory

Ghost-free galileon Lagrangians:

$\mathcal{L}_1^{(\mathrm{AdS})}$	(Wess-Zumino term)		$\frac{1}{4L^4}e^{4\phi}$
$\mathcal{L}_2^{(\mathrm{AdS})}$	$-\frac{1}{L^4}\sqrt{-g}$	$-\frac{1}{L^4}\sqrt{-g}$	$-\frac{1}{L^4}e^{4\phi}\sqrt{1+e^{-2\phi}L^2(\partial\phi)^2}$
$\mathcal{L}_3^{(\mathrm{AdS})}$	$\frac{1}{L^3}\sqrt{-g}K$	$\frac{1}{L^2}\sqrt{-g}\left[2\,S_1(\mathcal{D}\Lambda)+\frac{4}{L^2}\right]$	$\vdots$
$\mathcal{L}_4^{(\mathrm{AdS})}$	$-\frac{1}{L^2}\sqrt{-g}R$	$\sqrt{-g}\left[-8\,S_2(\mathcal{D}\Lambda)-\frac{12}{L^2}S_1(\mathcal{D}\Lambda)\right]$	
$\mathcal{L}_5^{(\mathrm{AdS})}$	$\frac{3}{2L}\sqrt{-g}\left[-\frac{1}{3}K^3+K^2_{\mu\nu}K-\frac{2}{3}K^3_{\mu\nu}-2\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\right)K^{\mu\nu}\right]$	$L^2\sqrt{-g}\left[48\,S_3(\mathcal{D}\Lambda)+\frac{48}{L^2}S_2(\mathcal{D}\Lambda)+\frac{18}{L^4}S_1(\mathcal{D}\Lambda)-\frac{12}{L^6}\right]$	

Weyl/AdS equivalence

Bellucci, Ivanov, Krivonos (2003)

Equate Maurer-Cartan forms:

$$V^{-1}dV = \omega_P^\alpha P_\alpha + \omega_D D + \omega_K^\alpha K_\alpha + \frac{1}{2}\omega_J^{\alpha\beta} J_{\alpha\beta} \ = \hat{\omega}_P^\alpha P_\alpha + \hat{\omega}_D D + \hat{\omega}_{\hat{K}}^\alpha \hat{K}_\alpha + \frac{1}{2}\hat{\omega}_J^{\alpha\beta} J_{\alpha\beta}$$

Invertible field re-definition:

$$y^\mu = x^\mu + L^2 e^{-\phi}\lambda^\mu, \quad e^\pi = \frac{e^\phi}{1+L^2\lambda^2}, \quad \xi^\mu = \lambda^\mu$$

$$x^\mu = y^\mu - L^2 \frac{e^{-\pi}}{1+L^2\xi^2}\xi^\mu, \quad e^\phi = \left(1+L^2\xi^2\right) e^\pi, \quad \lambda^\mu = \xi^\mu$$

Relation between invariant building blocks:

$$\sqrt{-g}d^D y = \det({e_\mu}^\alpha) d^D y \ = \det\left(\delta_\mu^\nu + L^2\mathcal{D}_\mu\Lambda^\nu\right)\sqrt{-\hat{g}}d^D x$$

$$\mathcal{D}\xi = \frac{1}{1+L^2\mathcal{D}\Lambda}\mathcal{D}\Lambda, \quad \mathcal{D}\Lambda = \frac{1}{1-L^2\mathcal{D}\xi}\mathcal{D}\xi$$

Weyl/AdS equivalence: galileons

Creminelli, Serone, Trincherini (2013)

Relation between galileon Lagrangians:

$$\begin{pmatrix} \mathcal{L}_1^{(\mathrm{AdS})} \\ \mathcal{L}_2^{(\mathrm{AdS})} \\ \mathcal{L}_3^{(\mathrm{AdS})} \\ \mathcal{L}_4^{(\mathrm{AdS})} \\ \mathcal{L}_5^{(\mathrm{AdS})} \end{pmatrix} = \begin{pmatrix} -1 & \frac{1}{2} & -\frac{1}{8} & \frac{1}{48} & -\frac{1}{384} \\ 4 & -1 & 0 & \frac{1}{24} & -\frac{1}{96} \\ -16 & 2 & 0 & \frac{1}{12} & -\frac{1}{24} \\ 0 & 12 & 0 & -\frac{1}{2} & 0 \\ 48 & -30 & 0 & -\frac{5}{4} & \frac{1}{8} \end{pmatrix} \begin{pmatrix} \mathcal{L}_1^{(\mathrm{Weyl})} \\ \mathcal{L}_2^{(\mathrm{Weyl})} \\ \mathcal{L}_3^{(\mathrm{Weyl})} \\ \mathcal{L}_4^{(\mathrm{Weyl})} \\ \mathcal{L}_5^{(\mathrm{Weyl})} \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{L}_1^{(\mathrm{Weyl})} \\ \mathcal{L}_2^{(\mathrm{Weyl})} \\ \mathcal{L}_3^{(\mathrm{Weyl})} \\ \mathcal{L}_4^{(\mathrm{Weyl})} \\ \mathcal{L}_5^{(\mathrm{Weyl})} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{8} & -\frac{5}{128} & \frac{1}{96} & -\frac{1}{384} \\ 0 & 0 & -\frac{1}{16} & \frac{1}{24} & -\frac{1}{48} \\ -8 & 0 & \frac{1}{8} & 0 & -\frac{1}{8} \\ 0 & 0 & -\frac{3}{2} & -1 & -\frac{1}{2} \\ 0 & -48 & -15 & -4 & -1 \end{pmatrix} \begin{pmatrix} \mathcal{L}_1^{(\mathrm{AdS})} \\ \mathcal{L}_2^{(\mathrm{AdS})} \\ \mathcal{L}_3^{(\mathrm{AdS})} \\ \mathcal{L}_4^{(\mathrm{AdS})} \\ \mathcal{L}_5^{(\mathrm{AdS})} \end{pmatrix}$$

Preserves vacuum, kinetic term → preserves S-matrix

$$-\frac{1}{2}\Lambda^2 e^{2\pi}(\partial\pi)^2 = -\frac{1}{16}\Lambda^2 L^2 \mathcal{L}_3^{(\mathrm{AdS})} + \frac{1}{24}\Lambda^2 L^2 \mathcal{L}_4^{(\mathrm{AdS})} - \frac{1}{48}\Lambda^2 L^2 \mathcal{L}_5^{(\mathrm{AdS})}$$

$$= -\frac{1}{2}\Lambda^2 e^{2\phi}(\partial\phi)^2 + \mathcal{O}\left(L^2\right)$$

Derivative expansion non-manifest in the AdS formulation

dS theory

Different parametrization of the coset: $\hat{K}^{\mu} = K^{\mu} - L^2 P^{\mu}$, $V = e^{x\cdot P} e^{\phi D} e^{\Lambda \cdot \hat{K}}$

invariant metric: $g_{\mu\nu} = e^{2\phi}\eta_{\mu\nu} - L^2 \partial_\mu \phi \partial_\nu \phi$

invariant building block:

$$\mathcal{D}_\mu \Lambda_\nu = \frac{1}{2L}\left(-K_{\mu\nu} + \frac{1}{L} g_{\mu\nu}\right)$$

$$K_{\mu\nu} = \frac{\gamma}{L}\left(e^{2\phi}\eta_{\mu\nu} + L^2 \partial_\mu \partial_\nu \phi - 2L^2 \partial_\mu \phi \partial_\nu \phi\right)$$

Lagrangians:

$$\mathcal{L}_2^{(\mathrm{dS})} = -\frac{1}{L^4} e^{4\phi} \sqrt{1 + e^{-2\phi} L^2 (\partial\phi)^2}$$

Weyl/dS equivalence

Equate Maurer-Cartan forms:

$$V^{-1}dV = \omega_P^\alpha P_\alpha + \omega_D D + \omega_K^\alpha K_\alpha + \frac{1}{2}\omega_J^{\alpha\beta} J_{\alpha\beta} \; = \hat{\omega}_P^\alpha P_\alpha + \hat{\omega}_D D + \hat{\omega}_{\hat{K}}^\alpha \hat{K}_\alpha + \frac{1}{2}\hat{\omega}_J^{\alpha\beta} J_{\alpha\beta}$$

Invertible field re-definition:

$$y^\mu = x^\mu - L^2 e^{-\phi}\lambda^\mu, \quad e^\pi = \frac{e^\phi}{1 - L^2\lambda^2}, \quad \xi^\mu = \lambda^\mu$$

$$x^\mu = y^\mu + L^2 \frac{e^{-\pi}}{1 - L^2\xi^2}\xi^\mu, \quad e^\phi = \left(1 - L^2\xi^2\right) e^\pi, \quad \lambda^\mu = \xi^\mu$$

relation between building blocks: $\sqrt{-g}d^D y = \det\left(\delta_\mu^\nu - L^2\mathcal{D}_\mu\Lambda^\nu\right)\sqrt{-\hat{g}}d^D x \quad , \quad \mathcal{D}\xi = \frac{1}{1 - L^2\mathcal{D}\Lambda}\mathcal{D}\Lambda, \quad \mathcal{D}\Lambda = \frac{1}{1 + L^2\mathcal{D}\xi}\mathcal{D}\xi$

relation among galileon Lagrangians:

$$\begin{pmatrix} \mathcal{L}_1^{(\mathrm{dS})} \\ \mathcal{L}_2^{(\mathrm{dS})} \\ \mathcal{L}_3^{(\mathrm{dS})} \\ \mathcal{L}_4^{(\mathrm{dS})} \\ \mathcal{L}_5^{(\mathrm{dS})} \end{pmatrix} = \begin{pmatrix} -1 & -\frac{1}{2} & -\frac{1}{8} & -\frac{1}{48} & -\frac{1}{384} \\ 4 & 1 & 0 & -\frac{1}{24} & -\frac{1}{96} \\ -16 & -2 & 0 & -\frac{1}{12} & -\frac{1}{24} \\ 0 & 12 & 0 & -\frac{1}{2} & 0 \\ 48 & 30 & 0 & \frac{5}{4} & \frac{1}{8} \end{pmatrix} \begin{pmatrix} \mathcal{L}_1^{(\mathrm{Weyl})} \\ \mathcal{L}_2^{(\mathrm{Weyl})} \\ \mathcal{L}_3^{(\mathrm{Weyl})} \\ \mathcal{L}_4^{(\mathrm{Weyl})} \\ \mathcal{L}_5^{(\mathrm{Weyl})} \end{pmatrix} \qquad \begin{pmatrix} \mathcal{L}_1^{(\mathrm{Weyl})} \\ \mathcal{L}_2^{(\mathrm{Weyl})} \\ \mathcal{L}_3^{(\mathrm{Weyl})} \\ \mathcal{L}_4^{(\mathrm{Weyl})} \\ \mathcal{L}_5^{(\mathrm{Weyl})} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{8} & -\frac{5}{128} & -\frac{1}{96} & -\frac{1}{384} \\ 0 & 0 & \frac{1}{16} & \frac{1}{24} & \frac{1}{48} \\ -8 & 0 & \frac{1}{8} & 0 & -\frac{1}{8} \\ 0 & 0 & \frac{3}{2} & -1 & \frac{1}{2} \\ 0 & -48 & -15 & 4 & -1 \end{pmatrix} \begin{pmatrix} \mathcal{L}_1^{(\mathrm{dS})} \\ \mathcal{L}_2^{(\mathrm{dS})} \\ \mathcal{L}_3^{(\mathrm{dS})} \\ \mathcal{L}_4^{(\mathrm{dS})} \\ \mathcal{L}_5^{(\mathrm{dS})} \end{pmatrix}$$

AdS/dS equivalence

Compose the two maps: $(\mathrm{AdS} \to \mathrm{Weyl}) \times (\mathrm{dS} \to \mathrm{Weyl})^{-1} = (\mathrm{AdS} \to \mathrm{dS})$

$$\begin{pmatrix} \mathcal{L}_1^{(\mathrm{dS})} \\ \mathcal{L}_2^{(\mathrm{dS})} \\ \mathcal{L}_3^{(\mathrm{dS})} \\ \mathcal{L}_4^{(\mathrm{dS})} \\ \mathcal{L}_5^{(\mathrm{dS})} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{1}{8} & 0 & \frac{1}{24} \\ 0 & 1 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{3}{2} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{15}{2} & 0 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} \mathcal{L}_1^{(\mathrm{AdS})} \\ \mathcal{L}_2^{(\mathrm{AdS})} \\ \mathcal{L}_3^{(\mathrm{AdS})} \\ \mathcal{L}_4^{(\mathrm{AdS})} \\ \mathcal{L}_5^{(\mathrm{AdS})} \end{pmatrix} \qquad \begin{pmatrix} \mathcal{L}_1^{(\mathrm{AdS})} \\ \mathcal{L}_2^{(\mathrm{AdS})} \\ \mathcal{L}_3^{(\mathrm{AdS})} \\ \mathcal{L}_4^{(\mathrm{AdS})} \\ \mathcal{L}_5^{(\mathrm{AdS})} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{1}{8} & 0 & \frac{1}{24} \\ 0 & 1 & 0 & -\frac{1}{6} & 0 \\ 0 & 0 & \frac{3}{2} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{15}{2} & 0 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} \mathcal{L}_1^{(\mathrm{dS})} \\ \mathcal{L}_2^{(\mathrm{dS})} \\ \mathcal{L}_3^{(\mathrm{dS})} \\ \mathcal{L}_4^{(\mathrm{dS})} \\ \mathcal{L}_5^{(\mathrm{dS})} \end{pmatrix}$$

Preserves vacuum, kinetic term ➡ preserves S-matrix

Amplitudes dS vs AdS

AdS 4-pt. amplitude:

$$\mathcal{L}^{(\mathrm{AdS})} = c_1\mathcal{L}_1^{(\mathrm{AdS})} + c_2\mathcal{L}_2^{(\mathrm{AdS})} + c_3\mathcal{L}_3^{(\mathrm{AdS})} + c_4\mathcal{L}_4^{(\mathrm{AdS})} + c_5\mathcal{L}_5^{(\mathrm{AdS})}$$

higher orders contribute

$$\begin{aligned}\mathcal{A}_4^{(\mathrm{AdS})} = \quad & \frac{1}{4Z}\left[1 + \frac{2}{Z}\left(c_3 - 6c_4 + 9c_5\right)\right] L^4(s^2+t^2+u^2) \\ & + \frac{3}{2Z^2}\left[c_4 - 4c_5 - \frac{1}{2Z}\left(c_3 - 6c_4 + 9c_5\right)^2\right] L^6 stu\end{aligned}$$

dS 4-pt. amplitude:

$$\mathcal{L}^{(\mathrm{dS})} = d_1\mathcal{L}_1^{(\mathrm{dS})} + d_2\mathcal{L}_2^{(\mathrm{dS})} + d_3\mathcal{L}_3^{(\mathrm{dS})} + d_4\mathcal{L}_4^{(\mathrm{dS})} + d_5\mathcal{L}_5^{(\mathrm{dS})}$$

$$\begin{aligned}\mathcal{A}_4^{(\mathrm{dS})} = \quad & \frac{1}{4Z}\left[-1 + \frac{2}{Z}\left(d_3 + 6d_4 + 9d_5\right)\right] L^4(s^2+t^2+u^2) \\ & + \frac{3}{2Z^2}\left[d_4 + 4d_5 - \frac{1}{2Z}\left(d_3 + 6d_4 + 9d_5\right)^2\right] L^6 stu\end{aligned}$$

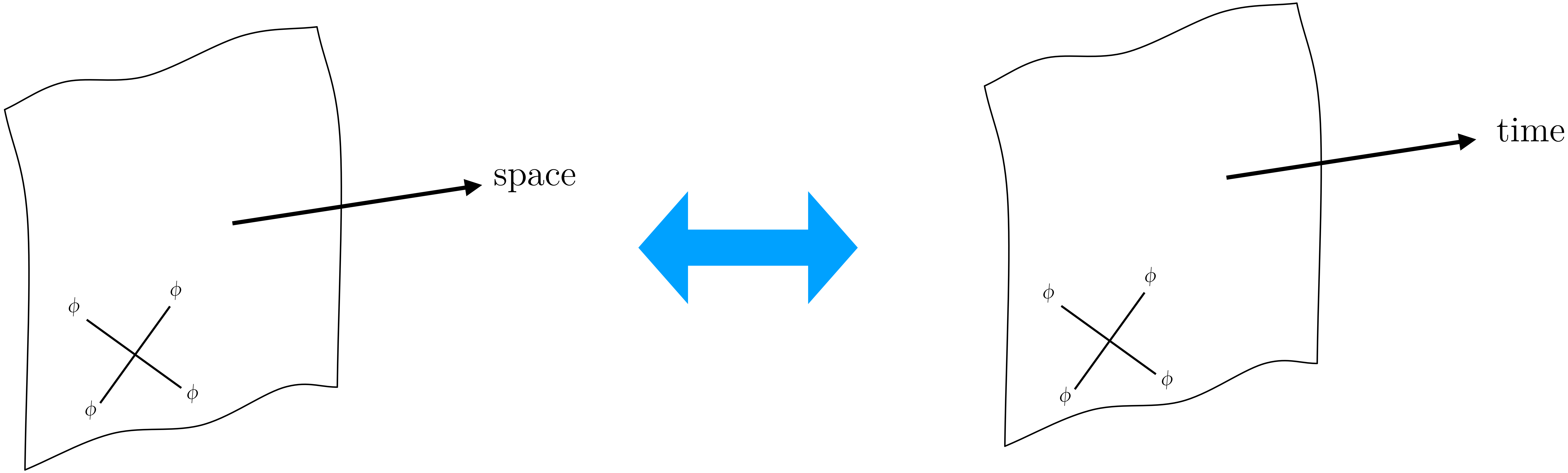
All amplitudes agree under the equivalence derived from the coset:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \frac{1}{8} & 0 & \frac{3}{2} & 0 & -\frac{15}{2} \\ 0 & -\frac{1}{6} & 0 & 1 & 0 \\ \frac{1}{24} & 0 & \frac{1}{6} & 0 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix}$$

Time vs. space

Typical question in AdS/CFT: how does CFT know whether to generate a spacelike or timelike holographic direction?

In our example: an observer stuck on the 3-brane cannot tell whether their brane is probing a spacelike or timelike extra dimension:



Conclusion likely changes if the bulk becomes dynamical in any way

How does all this extend to (A)dS branes and to higher spins?

Massive higher spin shift symmetries in (A)dS

James Bonifacio, KH, Austin Joyce, Rachel A. Rosen (1812.08167)

Massive spin s field on (A)dS:

$$\left(\Box - H^2\left[D + (s-2) - (s-1)(s+D-4)\right] - m^2\right)\ \phi_{\mu_1\cdots\mu_s} + \cdots = 0$$

$$m^2_{s,k} = -(k+2)(k+D-3+2s)H^2\,, \qquad k = 0, 1, 2, \ldots$$

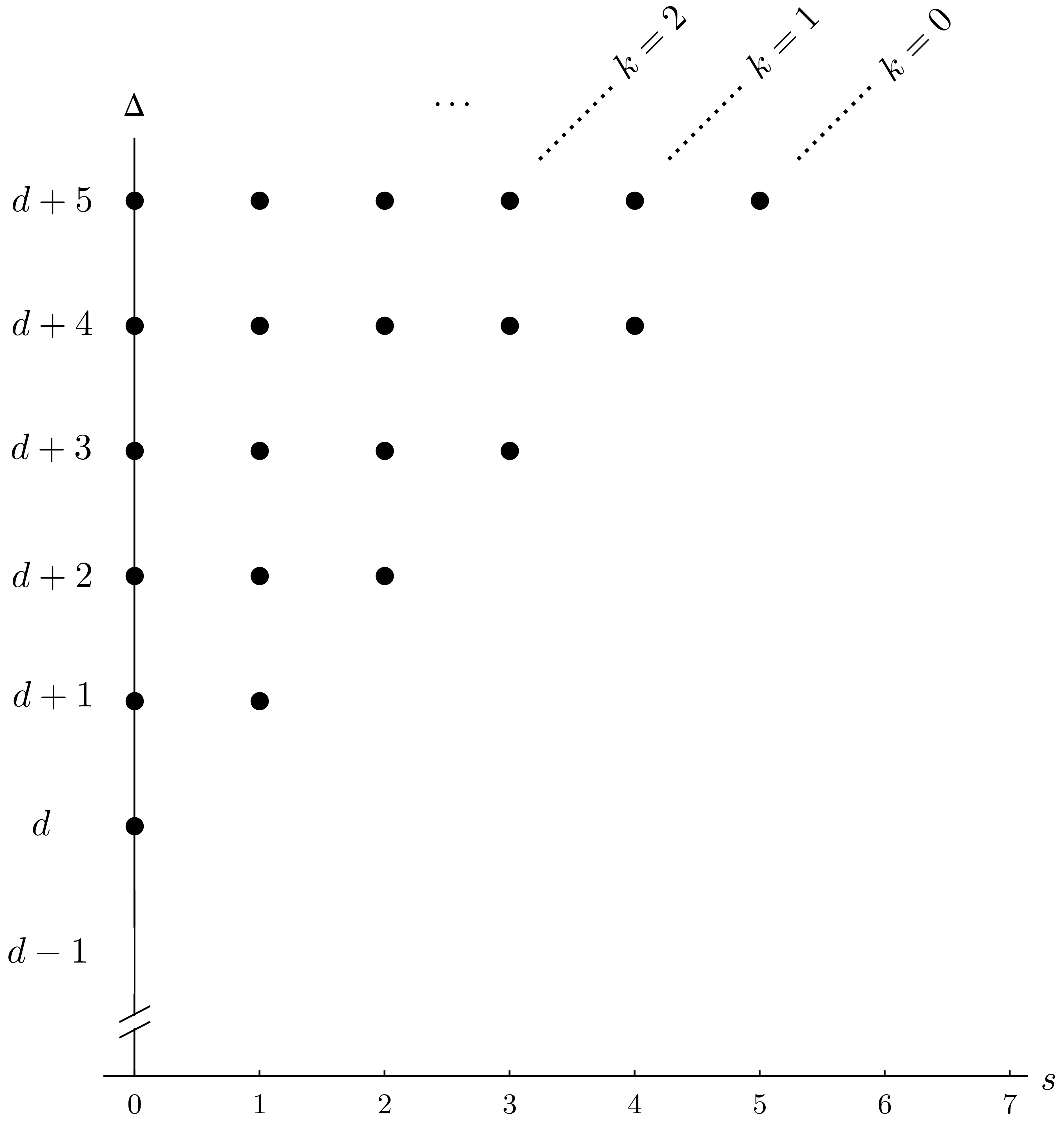
Symmetry under shifts parametrized by a mixed symmetry ambient space tensor:

$$\delta\phi_{\mu_1\ldots\mu_s} = S_{A_1\ldots A_{s+k},B_1\ldots B_s} X^{A_1}\ldots X^{A_{s+k}} \frac{\partial X^{B_1}}{\partial x^{\mu_1}} \cdots \frac{\partial X^{B_s}}{\partial x^{\mu_s}}\bigg|_{\text{(A)dS}}$$

$$S_{A_1\cdots A_{s+k},B_1\cdots B_s} \in \begin{array}{|c|c|}\hline \multicolumn{2}{|c|}{\quad s+k \quad} \\ \hline \multicolumn{1}{|c|}{s} & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}^{\,T}$$

Higher spins in (A)dS

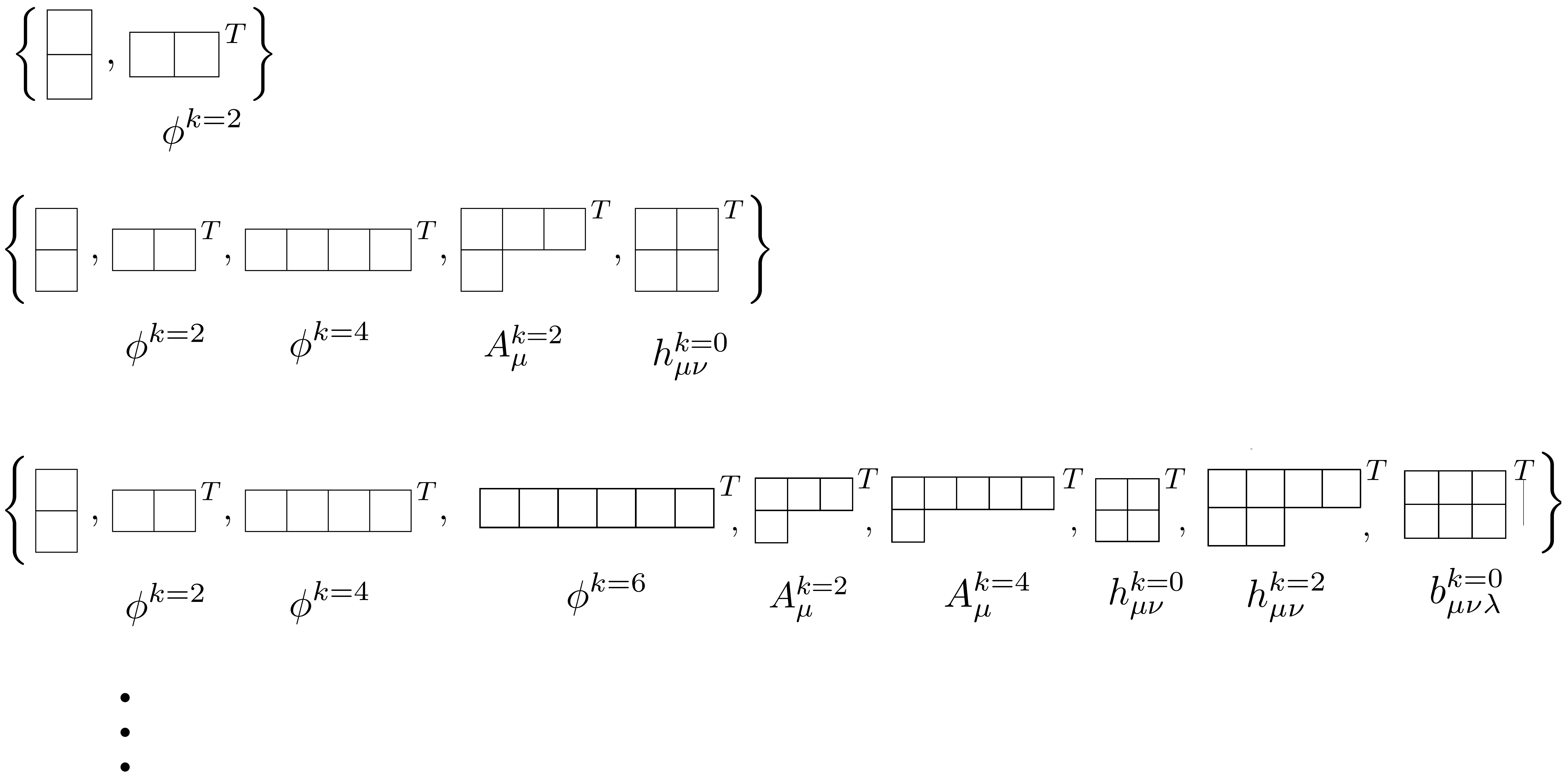
Dual CFT$_d$ operators: $\Delta = k + s + D - 1$



Other higher spin interactions?

There is a series of algebras which result from finite truncations of various higher spin algebras:

Boulanger, Skvortsov (2011)
Joung, Mkrtchyan (2015)



Is there a shift-symmetric theory with an infinite tower of fields coming from the longitudinal modes of Vasiliev theory?

Summary

- Power counting in EFTs coming from geometric setups can be subtle and obscured

- Gave an example where an EFT on a brane can't tell timelike from spacelike extra dimensions

- There should be other multi-field/higher-spin examples

- Possible interesting connection to novel AdS reps Basile, Joung, Oh (2023)